

## Nonperturbative Continuity in Graviton Mass versus Perturbative Discontinuity

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### Abstract

We address the question whether a graviton could have a small nonzero mass. The issue is subtle for two reasons: there is a discontinuity in the mass in the lowest tree-level approximation, and, moreover, the nonlinear four-dimensional theory of a massive graviton is not defined unambiguously. First, we reiterate the old argument that for the vanishing graviton mass the lowest tree-level approximation breaks down since the higher order corrections are singular in the graviton mass. However, there exist nonperturbative solutions which correspond to the summation of the singular terms and these solutions are continuous in the graviton mass.

Furthermore, we propose a nonlinear completion of the four-dimensional theory of massive gravity by embedding it into a five-dimensional generally covariant nonlinear model. We show that the exact solutions of the model are continuous in the mass, yet the perturbative expansion has the discontinuity in the leading order and the singularities in higher orders as in the four-dimensional case. The helicity-zero graviton state which is responsible for the perturbative discontinuity and singularities decouples from the matter in the limit of vanishing graviton mass in the full classical theory. Thus, we conclude that a small graviton mass is not in contradiction with observations.

# 1 Introduction

Could a graviton be massive? The naive answer to this question seems to be positive. Indeed, if the graviton Compton wavelength,  $\lambda_g = m_g^{-1}$ , is large enough, let us say of the present Hubble size, we should not be able to tell the massive graviton from a massless one. In fact, astrophysical bounds are even milder,  $\lambda_g > 10^{24}\text{cm}$  [1] (see also Refs. [2]). However, in general relativity (GR) the issue turns out to be more subtle. A dramatic observation has been made in Refs. [3, 4] according to which predictions of massless GR, such as the light bending by the Sun and the precession of the Mercury perihelion, differ by numerical factors from the predictions of the theory with a massive graviton, no matter how small the graviton mass is. This discontinuity, if true, would unambiguously prove that graviton is strictly massless in Nature.

The arguments of Refs. [3, 4] were based on the lowest tree-level approximation to interactions between sources. In this approximation the discontinuity has a clear physical interpretation. Indeed, a massive graviton in four-dimensions has *five* physical degrees of freedom (helicities  $\pm 2, \pm 1, 0$ ) while the massless graviton has only two (helicities  $\pm 2$ ). The exchange by the three extra degrees of freedom can be interpreted in the limit  $m_g \rightarrow 0$  as an additional contribution due to one massless vector particle with two degrees of freedom (“graviphoton” with helicities  $\pm 1$ ) plus one real scalar (“graviscalar” with the helicity 0). The graviphotons do not contribute to the one-particle exchange — their derivative coupling to the conserved energy-momentum tensor vanishes. The graviscalar, on the other hand, is coupled to the trace of the energy-momentum tensor and its contribution is generically nonzero. It is what causes the discontinuity between the predictions of massless and massive theory in the lowest tree-level approximation.

However, as was argued in Ref. [5], this discontinuity does not persist in the full classical theory. It was shown that the lowest tree-level approximation to the calculation of interactions between two sources breaks down when graviton mass is small. The next-to-leading terms in the corresponding expansion are huge since they are inversely proportional to powers of  $m_g$ . Thus, the truncation of the perturbative series does not make much sense and all higher order terms in the solution of classical equations for the graviton field should be summed up. The summation leads to nonperturbative solution which is continuous when  $m_g \rightarrow 0$ . The perturbative discontinuity shows up only at large distances where higher order terms are small, these distances are growing when  $m_g \rightarrow 0$ . In other words, the continuity is not perturbative and not uniform as a function of distance.

A simple reason why one could expect the violation of the lowest tree-level approximation is that it does not take into account the characteristic physical scale of the problem; while the nonperturbative calculation of the Schwarzschild solution does account for this effect. In the nonperturbative solution the coupling of the extra scalar mode to the matter is suppressed by the ratio of graviton mass to the physical scale of the problem. Hence, the predictions of the massive theory could be

made infinitely close to the predictions of the massless theory by taking small  $m_g$ .

To present these arguments in the form of a single equation let us consider the gravitational amplitude of scattering of a probe particle in the background gravitational field produced by a heavy static source. This amplitude can be presented in the form (note, that we use the flat metric  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ )

$$\tilde{h}^{\mu\nu}(q) t'_{\mu\nu} \propto \frac{a t^{\mu\nu} t'_{\mu\nu} - b t_\mu^\mu t_\nu'^\nu}{q^2 + m_g^2 - i\epsilon}, \quad (1)$$

where  $t_{\mu\nu} = p_\mu p_\nu$  and  $t'_{\mu\nu} = p'_\mu p'_\nu$  refer to the heavy particle with the four-momentum  $p_\mu = (M, \vec{0})$  and to the light particle with the momentum  $p'_\mu$  correspondingly <sup>1</sup>, see Fig. 1. The form factors  $a$  and  $b$  are functions of the momentum transfer  $q^2$  and are defined by two parameters: the graviton mass  $m_g$  and the Schwarzschild radius  $r_M = 2G_N M$  of the heavy particle with the mass  $M$ .

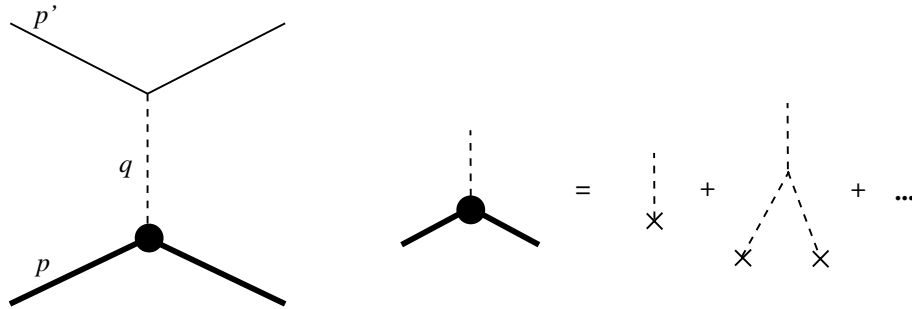


Figure 1: Scattering of the probe particle at the gravitational field of the heavy source. The bold circle accounts for summation of the higher order iterations over the nonlinearities in the classical equations.

In the lowest tree-level approximation of the massive theory  $a$  and  $b$  are just constants and the unitarity (sum over five helicities) fixes their ratio,  $a = 3b$ , while the same unitarity with two graviton states (helicities  $\pm 2$ ) in the massless theory gives  $a = 2b$ . Therefore, the discontinuity [3, 4] appears. However, this is only valid for the small momenta  $q \ll m_g (m_g r_M)^{-1/5}$  where the higher order corrections are small [5]. In the coordinate space it means that the linear approximation becomes valid only at the distance

$$r \gg r_m, \quad r_m \equiv \frac{(m_g r_M)^{1/5}}{m_g} = \frac{(2G_N M m_g)^{1/5}}{m_g}, \quad (2)$$

which for the Sun is bigger than the solar system size (see discussions in the next section).

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<sup>1</sup>To avoid the confusion note that we use  $t_{\mu\nu}$  only as a kinematical structure of the vertices not implying that it is the energy-momentum tensor.

On the other hand, at  $q \gg m_g (m_g r_M)^{-1/5}$ , i.e., at shorter distances,  $r \ll r_m$ , the summation of higher orders [5] returns the relation  $a = 2b$  of the massless theory. In other words, nonperturbative summation leads to decoupling of the graviscalar from the heavy source for distances  $r \ll r_m$ .

What was not verified in Ref. [5] is a matching of the nonperturbative solution at  $r \ll r_m$  with the exponentially decreasing linear solution at  $r \gg r_m$ . Boulware and Deser in [6] expressed their doubts about an existence of such a matching. Moreover, they argued that there is no consistent interacting theory of the massive spin 2 field in 3+1 dimensions. One of the arguments in Ref. [6] was that at quantum level the theory contained sixth state in addition to those five states. Moreover, the mass term in the action is not uniquely defined beyond the quadratic order in the fields.

These legitimate concerns can be addressed by embedding the model into a five-dimensional theory — a route we take in the present paper. Indeed, gravity in five dimension is well defined as a classical gauge theory, a massless graviton has exactly five states. For the matter fields which are confined to the four-dimensional brane the theory mimics the massive spin 2 particle with the fifth component of the momentum playing the role of the mass. Note the analogy with the supersymmetric BPS states whose mass is given by a central charge. This charge also can be viewed as an extra component of the momentum in the dimensionally enlarged space.

We analyze the issue of (dis)continuity within the framework of the five-dimensional model which allows for exactly solvable examples. In this case, the discontinuous perturbative result can be directly contrasted with the exact solutions, as opposed to the four-dimensional consideration [5] of the Schwarzschild problem in the massive theory. The model which we will discuss is that of Ref. [7]. It deals with a five-dimensional graviton which mimics the massive four-dimensional spin-2 state. In contrast with the 4-dimensional massive theory, in this case the full nonlinear action can be written. Based on the discussion above, we expect that in this model the perturbative discontinuity is absent in the full classical theory indeed. This can be checked by computing quantities in the lowest perturbative approximation and comparing them with the exact solutions of the Einstein equations. We do this comparison for the cosmological solutions found in Refs. [8, 9]. We show that the lowest tree-level perturbative result is off by a factor 4/3 as compared with the exact result and explain why the corresponding perturbation theory breaks down.

Note that recently the problem of the vanishing graviton mass was studied in a different setup. It was shown in Refs. [10] and [11] that there is no mass discontinuity even in the lowest tree-level exchange on de Sitter (dS) [12, 10] or Anti de Sitter (AdS) [10, 11] backgrounds<sup>2</sup>. This fits well with the discussions presented above. Indeed, in the case of the (A)dS background, even the lowest tree-level approximation does take into account the presence of a mass scale of the problem, which in that case is given by the cosmological constant  $\Lambda$ . It was shown in [11] that the coupling of graviscalar is proportional to  $m_g^2/\Lambda$  when  $m_g \rightarrow 0$  and deviations from the massless

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<sup>2</sup>The consideration for the dS space is a bit subtle since for  $m_g^2 < 2\Lambda/3$  ( $\Lambda$  being the cosmological constant) unitarity is violated in the theory [12].

model vanish in this limit. Since the cosmological constant in our world is restricted as  $\Lambda \leq 10^{-84} \text{ GeV}^2$ , this suppression would be operative for a graviton Compton wavelength which is bigger than the horizon size. In this case, as was pointed out in Ref. [10], the existence of such a tiny graviton mass becomes immaterial for all the practical purposes. Note also an interesting discussion of the continuity issue in the recent Ref. [13].

In Ref. [14] it was argued that in the (A)dS background the perturbative discontinuity reappears at the one-loop quantum level — the phenomenon very similar to the one-loop discontinuity for a massive non-Abelian vector fields discussed in [3]. This is certainly true since the loops are sensitive to the number of particles running in the loop diagrams. From the practical point of view, however, the comparison of the theory with the experimental data on the light bending by the Sun and the precession of the Mercury perihelion is not affected by the small quantum loop corrections. Indeed, while the graviscalar decouples from the classical source it is still coupled to the graviton and does contribute to the quantum loops. However, such effects of quantum gravity are suppressed and most likely cannot be disentangled in solar system measurements. For these reasons, in what follows we are focusing on the (dis)continuity in the classical theory only.

The paper is organized as follows. In section 2 we recall the essence of the graviton mass discontinuity found in Refs. [3, 4] and discuss the results of Ref. [5] where it was shown that there is in fact the continuity in the graviton mass in the full classical theory. In section 3 we introduce the five-dimensional nonlinear model which mimics the properties of a four-dimensional massive gravitational theory. We show that the perturbative discontinuity which is present in the lowest tree-level approximation disappears in the exact solution of the model. In section 4 we discuss another exact solution of the nonlinear model which interpolates between the four-dimensional and five-dimensional regimes. We conclude in section 5.

## 2 Preliminaries: Massive Graviton in 4D

We will consider the following action for a massive graviton on a flat 4D background:

$$S_m = M_{\text{Pl}}^2 \int d^4x \sqrt{|g|} \left( R + \frac{m_g^2}{4} [h_{\mu\nu}^2 - (h_\mu^\mu)^2] \right), \quad (3)$$

where  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  and the Planck mass  $M_{\text{Pl}}$  is related to the Newton constant  $G_N$  as  $M_{\text{Pl}}^2 = 1/(16\pi G_N)$ . The mass term has the Pauli-Fierz form [15], in quadratic in  $h_{\mu\nu}$  terms it is the only form which does not introduce ghosts [16]. We imply that indices in the mass term are raised and lowered by the tensor  $\eta_{\mu\nu}$ . If it were  $g_{\mu\nu}$  instead the difference would appear only in the cubic and higher in  $h_{\mu\nu}$  terms which are not fixed anyway; higher powers of  $h_{\mu\nu}$  could be arbitrarily added to the mass term.

In order to see the presence of the discontinuity in the lowest tree-level approximation let us compare free graviton propagators in the massless and massive theory. For the massless graviton we find:

$$D_{\mu\nu;\alpha\beta}^0(q) = \left( \frac{1}{2} \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{1}{2} \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{1}{2} \eta_{\mu\nu} \eta_{\alpha\beta} \right) \frac{1}{q^2 - i\epsilon}, \quad (4)$$

where only the momentum independent parts of the tensor structure is kept. By a gauge choice the momentum dependent structures can be taken zero. On the other hand, there is no gauge freedom for the massive gravity given by the action (3), and the propagator takes the following form:

$$D_{\mu\nu;\alpha\beta}^m(q) = \left( \frac{1}{2} \tilde{\eta}_{\mu\alpha} \tilde{\eta}_{\nu\beta} + \frac{1}{2} \tilde{\eta}_{\mu\beta} \tilde{\eta}_{\nu\alpha} - \frac{1}{3} \tilde{\eta}_{\mu\nu} \tilde{\eta}_{\alpha\beta} \right) \frac{1}{q^2 + m_g^2 - i\epsilon}, \quad (5)$$

where

$$\tilde{\eta}_{\mu\nu} = \eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_g^2}. \quad (6)$$

Note the  $1/m_g^4$ ,  $1/m_g^2$  singularities of the propagator.

The difference in the numerical coefficients for the  $\eta_{\mu\nu}\eta_{\alpha\beta}$  structure in the massless and massive propagators (1/2 versus 1/3) is what leads to the perturbative discontinuity [3, 4]. No matter how small the graviton mass is, the predictions are substantially different in the two cases. The structure (5) gives rise to contradictions with observations.

To see how this comes about let us calculate the amplitude of the lowest tree-level exchange by a graviton between two sources with energy-momentum tensors  $T_{\mu\nu}$  and  $T'_{\alpha\beta}$  (the sign tilde denotes the quantities which are Fourier transformed to momentum space):

$$\mathcal{A}_0 \equiv -8\pi G_N \tilde{T}_{\mu\nu} D_0^{\mu\nu;\alpha\beta} \tilde{T}'_{\alpha\beta} = -\frac{8\pi G_N}{q^2} \left( \tilde{T}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \tilde{T}^\beta_\beta \right) \tilde{T}'^{\mu\nu}. \quad (7)$$

In the massive case this amplitude takes the form:

$$\mathcal{A}_m \equiv -8\pi G_N \tilde{T}_{\mu\nu} D_m^{\mu\nu;\alpha\beta} \tilde{T}'_{\alpha\beta} = -\frac{8\pi G_N}{q^2 + m_g^2} \left( \tilde{T}_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} \tilde{T}^\beta_\beta \right) \tilde{T}'^{\mu\nu}. \quad (8)$$

In the relativistic normalization we are using  $\tilde{T}_{\mu\nu} = \langle p | T_{\mu\nu} | p \rangle = 2p_\mu p_\nu$  at zero momentum transfer,  $q = 0$ . Suppose we take two probe massive static sources with masses  $M_1$  and  $M_2$ . Then only  $\tilde{T}_{00}$ ,  $\tilde{T}'_{00}$  are non-vanishing and the lowest tree-level graviton exchange determines the Newtonian interaction,

$$\begin{aligned} V_0(r) &= \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\vec{r}} \frac{\mathcal{A}_0}{4M_1 M_2} = -\frac{G_N M_1 M_2}{r}, \\ V_m(r) &= \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\vec{r}} \frac{\mathcal{A}_m}{4M_1 M_2} = -\frac{4}{3} \frac{G_N M_1 M_2}{r} e^{-m_g r}. \end{aligned} \quad (9)$$

Expressions (7) and (8) give different results for the Newtonian attraction even in the range  $r \ll \lambda_g$  where one can neglect the exponential decrease. This difference can be eliminated by redefining the Newton coupling for the massive theory as follows:

$$\tilde{G}_N = \frac{4}{3} G_N , \quad (10)$$

where  $G_N$  is the Newton constant of the massless theory. For nonrelativistic problems the predictions of the massive theory with the coupling rescaled by a factor  $3/4$  at  $m_g \rightarrow 0$  are identical to those of the massless theory with the coupling  $G_N$ .

However, this is not enough to warrant the viability of the massive model. The relativistic predictions in the two cases are different [3,4]. For instance, the predictions for the light bending by the Sun are in conflict. At the classical level the trace of the energy-momentum tensor for light is zero. Therefore, the second term on the right hand side of Eqs. (7) and (8) is not operative for light. Hence, the amplitudes  $\mathcal{A}_0$  and  $\mathcal{A}_m$  are identical in this case. However, we have established above that calculations in the massive theory should be performed with the rescaled Newton constant. Taking into account this fact, the prediction for the light bending in the massive theory is off by 25 % [3,4].

We could certainly take an opposite point of view. Namely, do not rescale the Newton constant of the massive theory. In this case the predictions for the light bending in the massive and massless models are identical. However, the Newton force between static sources would differ by a factor of  $4/3$ .

The above considerations are based on the lowest perturbative approximation. The question is whether these results hold in the full classical theory. Normally, one would expect that for the solar system distances the lowest approximation is well justified. However, it was argued in Ref. [5] that the approximation breaks down in the massive theory for relatively short distances. Since this breaking manifests itself in a rather interesting way we will briefly summarize the results of Ref. [5] below.

To see the inconsistency of the perturbative expansion in  $G_N$  let us look (following [5]) at the Schwarzschild solution of (3). We parametrize the interval for a massive spherically symmetric body as follows:

$$ds^2 = -e^{\nu(\rho)} dt^2 + e^{\sigma(\rho)} d\rho^2 + e^{\mu(\rho)} \rho^2 (d\theta^2 + \sin^2\theta d\phi^2). \quad (11)$$

In the massless theory the function  $\mu$  is redundant due to the reparametrization invariance of the theory; it can be put equal to zero. However, in the massive case this gauge symmetry is broken and  $\mu$  is nonzero. Therefore, in order to compare the results in the massive and massless case one has to do the substitution:

$$r \equiv \rho \exp\left(\frac{\mu}{2}\right), \quad \exp(\lambda) \equiv \left(1 + \frac{\rho}{2} \frac{d\mu}{d\rho}\right)^{-2} \exp(\sigma - \mu). \quad (12)$$

The standard Schwarzschild solution of the massless theory takes the following form:

$$\begin{aligned}\nu^{\text{Schw}}(r) &= -\lambda^{\text{Schw}}(r) = \ln\left(1 - \frac{r_M}{r}\right) = -\frac{r_M}{r} - \frac{1}{2}\left(\frac{r_M}{r}\right)^2 + \dots, \\ \mu^{\text{Schw}}(r) &= 0.\end{aligned}\tag{13}$$

Here  $r_M \equiv 2G_N M$  is the gravitational radius of the source of mass  $M$ .

Let us compare this with the perturbative in  $G_N$  solution of the massive theory obtained in Ref. [5]. In the leading plus next-to-leading approximation in  $G_N$  the solution reads:

$$\begin{aligned}\nu &\simeq -\frac{r_M}{r} \left[ 1 + \frac{7}{32} \frac{r_M}{m_g^4 r^5} \right], \\ \lambda &\simeq \frac{1}{2} \frac{r_M}{r} \left[ 1 - \frac{21}{8} \frac{r_M}{m_g^4 r^5} \right], \\ \mu &\simeq \frac{1}{2} \frac{r_M}{m_g^2 r^3} \left[ 1 + \frac{21}{4} \frac{r_M}{m_g^4 r^5} \right].\end{aligned}\tag{14}$$

We note the following peculiarities of results (14):

- In the leading order there is the finite discontinuity in the expression for  $\lambda$ : The result of the massless theory in (2) differs from the result of the massive model by a factor  $1/2$ . This is precisely the discontinuity which is seen in the lowest approximation.
- The next-to-leading corrections in (14) are governed by the ratio  $r_M/m_g^4 r^5$  and are singular in  $m_g$ .
- For any given distance  $r$  there is a small values of  $m_g$  for which the perturbative expansion in  $G_N$  (more precisely in the corresponding dimensionful parameter) breaks down.

These results are in correspondence with perturbative series for the scattering amplitude described by Feynman graphs in Fig. 1. The leading terms in the expansions (14) are given by the diagram of the first order in the source, i.e., the diagram with one cross. The singular in  $m_g$  terms in the propagator (5) do not contribute in this order. In the next order (the diagram with two crosses in Fig. 1) we have two extra propagators which could provide a singularity in  $m_g$  up to  $1/m_g^8$ . Two leading terms  $1/m_g^8$  and  $1/m_g^6$  do not contribute again so the result contains only the  $1/m_g^4$  singularity as in Eq. (14).

To demonstrate how badly the expansion in powers of  $G_N$  breaks down let us take the largest allowed value for the graviton mass,  $m_g = (10^{25} \text{ cm})^{-1}$  [1, 2] and calculate the correction to the leading result in the gravitational field of the Sun. We will find that at distances of order of the solar system size, i.e., at  $r \sim 10^{15} \text{ cm}$ , the next-to-leading corrections in (14) are about  $10^{32}$  times bigger than the leading terms. Therefore, this expansion is unacceptable.



For a light enough graviton, however, a consistent perturbative expansion could be organized in powers of  $m_g$ . In this case one finds [5]:

$$\begin{aligned}\nu(r) &= -\frac{r_M}{r} + \mathcal{O}\left(m_g^2 \sqrt{r_M r^3}\right), & \lambda(r) &= \frac{r_M}{r} + \mathcal{O}\left(m_g^2 \sqrt{r_M r^3}\right), \\ \mu(r) &= \sqrt{\frac{8r_M}{13r}} + \mathcal{O}\left(m_g^2 r^2\right),\end{aligned}\tag{15}$$

where only the leading terms in  $r_M/r$  are retained. The expansion in  $m_g$  is applicable in the following interval:

$$r_M \ll r \ll r_m, \quad r_M = 2G_N M, \quad r_m = \frac{(m_g r_M)^{1/5}}{m_g}.\tag{16}$$

For the gravitational field of the Sun this would correspond to the interval:

$$3 \cdot 10^5 \text{ cm} \ll r \ll 10^{21} \text{ cm},\tag{17}$$

where the lower bound is less than the radius of the Sun and the upper bound is of the order of a galaxy scale. Thus, for practical calculations within the solar system this expansion is well suited.

As we see, the expressions for  $\mu$  and  $\lambda$  in the leading approximation coincide with those of the massless theory (2). Thus, there is *no mass discontinuity*. Moreover, the expressions (15) explicitly shows non-analyticity in  $G_N$ ,  $\mu \propto \sqrt{G_N}$ , for  $\nu$  and  $\lambda$  non-analytic terms are proportional to  $m_g^2$ .

There is one subtle issue concerning the validity of the results discussed above. In this approach it is complicated to make sure that the solutions which have no discontinuity do indeed satisfy the boundary conditions at infinity, i.e., that for  $r \gg 1/m_g$  the solution matches the exponentially decreasing function.

In the next section we present the model which is free of this disadvantage. We perform our analysis in a completely nonlinear theory in which exact solutions can be found. These solutions are compared with the perturbative results. We find that the picture outlined in the work [5] (and discussed above) holds.

### 3 A Brane Model of Massive Graviton

The 5D model which we will discuss was introduced in [7]. The gravitational part of the action takes the form:

$$S = M_*^3 \int d^4x dy \sqrt{|G|} \mathcal{R} + M_{\text{Pl}}^2 \int d^4x \sqrt{|g|} R(x).\tag{18}$$

Where  $M_*$  is a parameter of the theory and  $M_{\text{Pl}} = 1.7 \cdot 10^{18} \text{ GeV} \gg M_*$ . Furthermore,  $G_{AB}$  is 5D metric tensor,  $A, B = 0, 1, \dots, 4$ , and  $\mathcal{R}$  is the five-dimensional Ricci scalar,  $g_{\mu\nu}$  denotes the induced metric on the brane which we take as

$$g_{\mu\nu}(x) \equiv G_{\mu\nu}(x, y=0), \quad \mu, \nu = 0, 1, 2, 3,\tag{19}$$

neglecting the brane fluctuations.

We assume that our observable 4D world (4D matter) is confined to a tensionless brane (a tensionless hyper-plane in this case) which is fixed at the point  $y = 0$  in extra fifth dimension<sup>3</sup>. In other words, we assume that the energy-momentum tensor of 4D matter has the following factorized form  $T_{\mu\nu}(x) \delta(y)$ . We also imply the presence of the Gibbons-Hawking boundary term on the brane, this provides the correct Einstein equations in the bulk. These simplifications help to keep the presentation clear and do not affect our main results. The brane world aspects of the model (18) were studied in detail in Refs. [7, 17–19].

Let us study the gravitational potential between two static bodies located on the brane. This can be calculated from the action (18). The corresponding Green function is conveniently represented by working in momentum space in the four world-volume directions and in position space with respect to the transverse coordinate  $y$ . The Green function between two points on the brane is given by [7]:

$$\tilde{G}(p, y = 0) = \frac{1}{M_{\text{Pl}}^2} \frac{1}{p^2 + m_c \sqrt{p^2}}, \quad (20)$$

where we introduce the parameter

$$m_c \equiv \frac{1}{r_c} \equiv \frac{2 M_*^3}{M_{\text{Pl}}^2}. \quad (21)$$

The Green function (20) has unusual features. It has a tachyonic pole at  $p^2 = -p_0^2 + \vec{p}^2 = m_c^2$  which corresponds to the decay into the continuous tower of Kaluza-Klein states (which arise from the reduction of 5D graviton). Although, the five-dimensional graviton is well defined, from the 4D perspective it looks as unstable particle with the width  $m_c$ . Nevertheless, the rules of integration for the propagator (20) in the complex energy plane can be defined consistently.

In particular, using (20) we can find the static potential  $\phi(r)$ . The result can be written in terms of special functions and has different asymptotic behavior for small and large distances (see Ref. [7]). The “crossover scale” between these two regimes is defined by  $r_c$  given in Eq. (21). At short distances, i.e., when  $r \ll r_c$

$$\phi(r) = -\frac{1}{8\pi^2 M_{\text{Pl}}^2} \frac{1}{r} \left\{ \frac{\pi}{2} + \left[ -1 + \gamma - \ln\left(\frac{r_c}{r}\right) \right] \left(\frac{r}{r_c}\right) + \mathcal{O}(r^2) \right\}. \quad (22)$$

Here  $\gamma \simeq 0.577$  is the Euler constant. The leading term in this expression has the familiar  $1/r$  scaling of the four-dimensional Newton law with a right numerical coefficient. The leading correction is given by the logarithmic *repulsion* term in (22).

Let us turn now to the large distance behavior. For  $r \gg r_c$  one finds:

$$\phi(r) = -\frac{1}{16\pi^2 M_*^3} \frac{1}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right). \quad (23)$$

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<sup>3</sup>A simplest possibility is to consider a brane at a fixed point of the  $\mathbf{R}/\mathbf{Z}_2$  orbifold.

The long distance potential scales as  $1/r^2$  in accordance with the 5D Newton law. Thus, the crossover scale (21) should be sufficiently large to avoid conflict with astronomical observations. In [7] it was estimated that for  $M_* \sim 1$  TeV, the crossover scale  $r_c$  is around  $10^{15}$  cm, which is roughly the size of the solar system. This is too low to be consistent with data. Therefore, the scale  $M_*$  should be taken to be at least a couple of orders smaller than 1 TeV. This is in no conflict with any gravitational or Standard Model measurements (see discussions in Ref. [18, 19]). We take  $r_c \geq 10^{25}$  cm which corresponds to  $M_* \leq 1$  GeV.

The parameter  $m_c$  plays the role in this model which in many respects is similar to that of the graviton mass  $m_g$  in (3). Indeed, as  $m_c \rightarrow 0$ , gravity on a brane becomes 4D Newtonian at more and more larger distances. Moreover, the four-dimensional interaction in the model with the action (18) can be interpreted as an exchange of a four-dimensional state with the width equal to  $m_c$  [7]. In the next section we will find even more closer similarities between  $m_c$  and  $m_g$ .

### 3.1 Perturbative Discontinuity

To see that the model (18) exhibits the discontinuity in the one-graviton tree-level approximation let us calculate, following [7], the tensorial structure of one-graviton exchange. To this end we will solve the Einstein equations in the linear approximation in  $h_{AB}$  which is the deviation from the flat 5D metric,

$$G_{AB} = \eta_{AB} + h_{AB} . \quad (24)$$

We choose the *harmonic gauge* in the bulk:

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h_C^C . \quad (25)$$

In this gauge from the  $\{\mu 5\}$  and  $\{55\}$  components of the sourceless equations of motion follows that

$$h_{\mu 5} = 0, \quad h_5^5 = h_\mu^\mu . \quad (26)$$

Let us turn to the  $\{\mu\nu\}$  components of the Einstein equations. After some simplifications they take the form:

$$\left( M_*^3 \partial_A \partial^A + M_{\text{Pl}}^2 \delta(y) \partial_\alpha \partial^\alpha \right) h_{\mu\nu} = - \left\{ T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T_\alpha^\alpha \right\} \delta(y) + M_{\text{Pl}}^2 \delta(y) \partial_\mu \partial_\nu h_\alpha^\alpha . \quad (27)$$

There are two terms on the right hand side of this equation. The first one has a structure which is identical to that of a massive 4D graviton (or, equivalently of a massless 5D graviton). The second term on the right hand side which contains derivatives  $\partial_\mu \partial_\nu$  is not important at the moment since it vanishes when it is contracted with the conserved energy-momentum tensor. As a result, the amplitude of interaction of two test sources takes the form:

$$\tilde{h}_{\mu\nu}(p, y=0) \tilde{T}'^{\mu\nu}(p) \propto \frac{\tilde{T}^{\mu\nu} \tilde{T}'_{\mu\nu} - \frac{1}{3} \tilde{T}_\mu^\mu \tilde{T}'_\nu{}^\nu}{p^2 + m_c p} , \quad (28)$$

where  $p \equiv \sqrt{p^2}$ . We see that the tensor structure is the same as in the case of the massive 4D theory, see Eq. (8).

In analogy with the discussions in the previous section we could expect that the lowest tree-level approximation will break down in the next iterations in the classical source. Further indication on this is the existence of the singular in  $m_c$  terms in the expression for the gravitational field  $h_{\mu\nu}$  produced by a static source. We write the energy-momentum tensor for the source as follows:

$$T_{\mu\nu}(x) = -M \delta_{\mu 0} \delta_{\nu 0} \delta^{(3)}(\vec{x}), \quad (29)$$

where  $M$  is its rest mass. As before, let us make Fourier transform with respect to four world-volume coordinates. Then the solution looks as follows:

$$\tilde{h}_{00}(p, y) = c \frac{2}{3} G_N M \frac{1}{p^2 + m_c p} \exp(-p|y|), \quad (30)$$

$$\begin{aligned} \tilde{h}_{ij}(p, y) &= c \frac{1}{3} G_N M \frac{\delta_{ij}}{p^2 + m_c p} \exp(-p|y|) \\ &+ c \frac{4}{3} G_N M \frac{p_i p_j}{m_c p} \frac{1}{p^2 + m_c p} \exp(-p|y|), \end{aligned} \quad (31)$$

where  $c = -16\pi$ . These expressions, taken at  $y = 0$ , should be contrasted with the lowest order expressions for the Schwarzschild solution in 4D theory with a massless graviton:

$$\tilde{h}_{00}^{\text{Schw}}(p) = c \frac{2}{3} G_N M \frac{1}{p^2}, \quad (32)$$

$$\tilde{h}_{ij}^{\text{Schw}}(p) = c \frac{2}{3} G_N M \frac{\delta_{ij}}{p^2}. \quad (33)$$

Comparing the expressions (30-31) to those in (32-33) we draw the following conclusions:

- The  $\{00\}$  components coincide for large momenta, or, equivalently for  $r \ll r_c$ .
- The  $\{ij\}$  component of the 5D theory consists of two terms. The first term is twice as small as the corresponding term on the right hand side of the Schwarzschild solution (33). This is what gives rise to the discontinuity in the free propagator.
- There is an additional term in the expression for  $\tilde{h}_{ij}(p, y = 0)$  which is proportional to:

$$\frac{p_i p_j}{m_c p}.$$

This term does not contribute to the one-graviton exchange in the leading order because of conservation of the energy-momentum tensors (the diagram

with a single cross in Fig. 1). However, it does contribute to higher order diagrams (the ones with two and more crosses in Fig. 1). This term is singular in  $m_c$  and the perturbation theory in  $G_N$  breaks down when  $m_c \rightarrow 0$ .

Given these arguments, we conclude that for a consistent calculation of the interaction between two sources on a brane we should find the Schwarzschild solution which sums up all the orders of the Born expansion for the classical equations. Unfortunately, we could not manage to find the analytic solution. However, implying the existence of a smooth in  $m_c \rightarrow 0$  limit, one could perform the expansion in  $m_c$  in analogy with the 4D massive case [5].

The  $\{\mu\nu\}$  component of the Einstein equation for the action (18) can be integrated with respect to  $y$  in the interval  $-\epsilon \leq y \leq \epsilon$  with  $\epsilon \rightarrow 0$ . The resulting equation takes the form:

$$\bar{\mathcal{G}}_{\mu\nu}(x) + m_c \int_{-\epsilon}^{+\epsilon} \mathcal{G}_{\mu\nu}(x, y) dy = -\frac{M}{2 M_{\text{Pl}}^2} \delta_{\mu 0} \delta_{\nu 0} \delta^{(3)}(x), \quad (34)$$

where  $\bar{\mathcal{G}}_{\mu\nu}$  and  $\mathcal{G}_{\mu\nu}(x, y)$  denote the Einstein tensor of the worldvolume and bulk theories respectively. Since the extrinsic curvature has a finite jump across the brane, the second term on the left hand side of (34) is nonzero even in the limit  $\epsilon \rightarrow 0$ . This term is proportional to the parameter  $m_c$  with respect to which the expansion is performed (we imply that the metric is nonsingular in  $m_c$ , this seems to be a reasonable requirement for a physically meaningful solution).

Then, it is clear from (34) that in the lowest approximation in  $m_c$  one recovers the usual 4D Schwarzschild solution of the massless theory (2). For the calculation of the sub-dominant corrections in  $m_c$  and for matching conditions at infinity, however, numerical simulations are needed. Note that in this case the solution should be matched at infinity to a well known 5D Schwarzschild solution which decreases as  $(r_M/r)^2$  at infinity. This is an easier task compared to the 4D massive case where the power-law solution at short distances should be matched with the Yukawa potential at infinity<sup>4</sup>.

Does this mean that we cannot compare analytically the perturbative and non-perturbative results in the model (18)? Not at all. Instead of finding the exact Schwarzschild solution we perform the similar analysis for other solutions which can be obtained explicitly. In the next section we discuss an exact nonperturbative cosmological solution of the model (18) found in Refs. [8, 9] which differs from the perturbative result by  $4/3$ .

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<sup>4</sup>The next morning after this paper was submitted to the archive an interesting work [20] appeared. In Ref. [20] the asymptotic form of the Schwarzschild solution for  $m_c \rightarrow 0$  was also discussed and, moreover, certain generalizations of cosmological solutions of the model (18) were obtained.

### 3.2 Nonperturbative continuity

In this section we study the cosmological solution in the model (18) found in Ref. [8]. It was already noticed in [8] that the cosmological evolution in (18) is governed by the Newton constant which differs from the the “Newton” constant of perturbation theory by  $4/3$ . We will discuss in details this discrepancy.

Our goal is as follows. We consider the solution of the model (18) which describes the expansion of the matter dominated Universe. We will perform two distinct calculation for this. First we find the solution based on the Newtonian approximation. This calculation makes use of the lowest order potential between objects on the brane. As a second step we find the corresponding exact nonperturbative cosmological solution of the Einstein equations. In the domain where the Newtonian approximation is legitimate, the perturbative result for the cosmological solution would coincide under the normal circumstances with the nonperturbative one as it happens in 4D world with a massless graviton. However, we find the discrepancy by a factor of  $4/3$  in these two methods.

Let us start with the perturbative approach. As we established in the previous subsection the one-graviton exchange in the lowest approximation gives rise to the following expression for the potential of a massive source at short distances  $r \ll r_c$ :

$$\phi(r) = -\tilde{G}_N \frac{M}{r}. \quad (35)$$

The appearance of the constant  $\tilde{G}_N$  instead of  $G_N$  in this expression is related to the fact that we used the lowest tree-level approximation.

Let us now use the standard consideration of the Newtonian cosmology<sup>5</sup>. Consider a spherical ball with some uniform matter density in it. We assume that the radius of the ball  $R$  is much smaller then  $r_c$  and that we are in a regime where the Newtonian approximation is valid. In this case the potential of the ball on its surface takes the form:

$$\phi_{\text{ball}}(r = R) = -\tilde{G}_N \frac{M}{R}. \quad (36)$$

Let us consider a point-like probe particle of mass  $m_0$  which is located just right on the surface of the ball. We neglect the back-reaction of this probe particle on the ball. The energy conservation condition for the system of the ball and probe takes the form:

$$\frac{m_0 \dot{R}^2}{2} - \tilde{G}_N \frac{M m_0}{R} = \frac{k m_0}{2}, \quad (37)$$

where dots denote the time derivative and  $k$  is some constant. We would like to calculate the time evolution of the radius  $R$ . In the regime which we discuss this is equivalent to the time evolution of the scale factor in Friedmann-Lemaitre-

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<sup>5</sup>For a careful treatment and interpretation of the Newtonian cosmology see, e.g., [21].

Robertson-Walker cosmology. In what follows we consider the solution which corresponds to the expansion of flat, i.e.,  $k = 0$ , matter-dominated Universe. For  $k = 0$  we rewrite (37) as follows:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} \tilde{G}_N \rho, \quad (38)$$

where the density  $\rho$  for the matter-dominated Universe is related to the scale factor  $R$  as follows

$$\rho = \frac{u}{R^3}, \quad (39)$$

where  $u$  is some constant.

This is nothing but the Friedmann equation for the scale factor  $R$  for a flat matter-dominated Universe. We find the solution for the scale factor:

$$R^3(t) = 6\pi u \tilde{G}_N t^2. \quad (40)$$

This solution is consistent with the fact that we choose the time period when  $R \ll r_c$  so that the brane world evolves in accordance with laws of 4D theory. What is important in our solution is the numerical coefficient in the relation (40) which we will compare with the exact solution below.

Let us now solve the same problem using the exact Einstein equations. We parametrize the 5D interval in the following form:

$$ds^2 = -N^2(t, y) dt^2 + A^2(t, y) dx_i dx^i + B^2(t, y) dy^2. \quad (41)$$

The 4D scale factor is defined as follows:

$$R(t) \equiv A(t, y = 0). \quad (42)$$

The solution was found in [8]:

$$N = 1 - |y| \frac{\ddot{R}}{\dot{R}}, \quad A = R - |y| \dot{R}, \quad B = 1, \quad (43)$$

and the 4D scale factor obeys the following modified Friedmann equation:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho + m_c \frac{\dot{R}}{R}. \quad (44)$$

After introducing the variable

$$x \equiv 1 + \frac{32\pi G_N}{3m_c^2} \rho, \quad (45)$$

the solution for the matter-dominated epoch takes the form:

$$\frac{3m_c}{2} t = -\frac{\sqrt{x}}{1-x} + \frac{1}{2} \log \left| \frac{1+\sqrt{x}}{1-\sqrt{x}} \right| + \frac{1}{x-1}. \quad (46)$$

We notice the following properties of the exact solution:

- If we expand the solution in  $G_N$ , more precisely in  $G_N \rho/m_c^2$ , the higher order terms are singular in  $m_c$ .
- Since  $\rho \sim 1/G_N$ ,  $G_N$  cannot serve as an appropriate expansion parameter.
- When  $m_c \rightarrow 0$  the useful expansion parameter is  $1/x$ .

Indeed, in the limit of small  $m_c$

$$m_c \rightarrow 0, \quad x \rightarrow \frac{32\pi G_N}{3m_c^2} \rho, \quad (47)$$

and expanding in powers of  $1/x$  we find from (46):

$$R^3(t) = 6\pi u G_N t^2 = \frac{9}{2} \pi u \tilde{G}_N t^2. \quad (48)$$

This should be compared with the perturbative result (40). As we see, Eq. (40) which was obtained using the lowest tree-level approximation is off the exact result (48) by the same factor  $4/3$ .

## 4 An Interpolating Solution

In this section we discuss a cosmological solution found in [9] and show that it interpolates between the regimes with 4D and the 5D tensor structures.

Let us start with the brane action (18) and in addition introduce in the theory a negative cosmological constant on the brane  $\Lambda_b$  and the matter density  $\rho \geq |\Lambda_b|$  (we put pressure equal to zero for simplicity). The time evolution of such a 4D brane universe is interesting, it evolves asymptotically to a static Minkowski space on the brane without any fine tuning [9]. The asymptotic form of the metric is as follows:

$$ds^2 = -(1 + b|y|)^2 dt^2 + dx^i dx_i + dy^2, \quad (49)$$

with the constant

$$b \equiv |\Lambda_b|/4M_*^3. \quad (50)$$

In fact, this is a solution to the equation

$$\mathcal{R}_{AB} - \frac{1}{2} G_{AB} \mathcal{R} = \frac{1}{2M_*^3} T_{AB}(x) \delta(y), \quad (51)$$

where the energy-momentum tensor on the brane is

$$T_{\mu\nu} = \text{diag}(0, -\Lambda_b, -\Lambda_b, -\Lambda_b), \quad T_{5\mu} = T_{55} = 0, \quad (52)$$

and in addition, to warrant the 4D behavior, we include the induced term on the brane [9].



The important thing is that the early cosmology of this model is standard, with no discontinuity in the Newton constant. Indeed, the Friedmann equation is given in (44) where  $\rho$  should be substituted by  $\rho + \Lambda_b$ . The Newton constant on the right hand side of this equation is the conventional 4D gravitational constant which reflects *no discontinuity*. This is true as long as the early cosmology is concerned.

Let us now look at the late cosmology, more precisely at the form of the metric (49) to which the solution asymptotes. The metric on the brane is Minkowskian and static everywhere with only dependence on  $y$ . For small values of  $y$ , which satisfy  $b|y| \ll 1$ , this metric can be obtained as a perturbation on the flat Minkowski space. Indeed, for small perturbations (24) in the harmonic gauge (25) we find Eq. (27) with the energy momentum tensor defined in (52). This equation has the 5D tensor structure on the right hand side. Let us now notice that the energy-momentum tensor (52) satisfies the relation:

$$T_{ij} - \frac{1}{3} T \eta_{ij} = 0 . \quad (53)$$

Therefore, the equation for  $h_{ij}$  is simplified. This is completely due to the 5D tensor structure; in fact had we have a 4D tensor structure, this would not be so. Furthermore, the solution of equation (27) in the gauge (25) can be written in the following form:

$$h_{00} = -h_{55} = -\frac{|\Lambda_b|}{2 M_*^3} |y| , \quad h_{ij} = 0 , \quad h_{\mu 5} = 0 . \quad (54)$$

One can indeed verify that this solution coincides in the first order with the exact solution (49). For this we perform the following gauge transformation of the exact solution (two different signs correspond to the two sides of the brane):

$$y = \text{sign}(z) \frac{1}{b} \left[ \left( 1 + 2b|z| + 2b^2 z^2 \right)^{1/2} - 1 \right] . \quad (55)$$

After this the metric takes the form

$$ds^2 = -(1 + 2b|z| + 2b^2 z^2) dt^2 + dx^i dx_i + \frac{(1 + 2b|z|)^2}{1 + 2b|z| + 2b^2 z^2} dz^2 , \quad (56)$$

which in the leading order coincides with the perturbative solution.

Therefore, we conclude that the cosmological solution of Ref. [9] does indeed provide an explicit example with both asymptotic regimes: the small distance (small Hubble radius) behavior is 4-dimensional with the 4D tensor structure, whereas the large distance (large Hubble radius) behavior has the 5D tensor structure. In this sense the solution discussed above captures all the features of a Schwarzschild solution of 4D massive theory; this is not surprising since it is asymptotically (in time) Minkowski on the brane.

## 5 Discussions and Conclusions

We proposed a nonlinear completion of a four-dimensional theory of massive gravitation by embedding it into a five-dimensional generally covariant nonlinear classical theory. The mass discontinuity is present in the lowest tree-level approximation, however, this approximation breaks down for the vanishing graviton mass and all the tree-level graphs should be taken into account. The resulting expression of the nonperturbative classical calculation is continuous in the graviton mass. Thus, there is no mass discontinuity in the full classical theory.

There are three extra degrees of freedom in the massive (or 5-dimensional) theory compared to the massless one. Among these degrees of freedom only the helicity 0 state (the graviscalar) has a nonzero coupling to 4D matter. However, this coupling tends to zero in full classical theory as the graviton mass (or  $m_c$  in the 5D example) vanishes. Thus, all the extra degrees of freedom decouple in the massless limit.

The interesting issue which we did not discuss in the paper is the emission of a helicity 0 gravitons. Based on our observations and using the unitarity arguments we expect that the nonperturbative amplitudes of the radiation of the helicity 0 state by 4D matter fields also vanish with the graviton mass, while they are nonvanishing in the lowest tree-level approximation as was shown in Ref. [22].

In the small mass limit the extra degrees of freedom of a massive theory form an independent sector which decouples from our matter as the graviton mass goes to zero. These degrees of freedom do interact with each other; moreover, in perturbation theory these interactions are singular in the limit  $m_g \rightarrow 0$ . Certainly, on top of the classical effects there is an issue of quantum loops which we did not discuss in the present work. However, the loop effects are suppressed and most likely they cannot be disentangled in existing measurements.

Therefore, based on the current gravitational experiments we are not able to tell a massless graviton from a very light massive graviton.

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## References

- [1] D. E. Groom *et al.* [Particle Data Group Collaboration],  
Eur. Phys. J. C **15**, 1 (2000).

- [2] A. S. Goldhaber and M. M. Nieto, Phys. Rev. D **9** (1974) 1119;  
C. M. Will, Phys. Rev. D **57**, 2061 (1998) [gr-qc/9709011];  
J. Uzan and F. Bernardeau, hep-ph/0012011;  
P. Binetruy and J. Silk, astro-ph/0007452.
- [3] H. van Dam and M. Veltman, Nucl. Phys. **B22**, 397 (1970) .
- [4] V.I. Zakharov, JETP Lett. **12**, 312 (1970) .
- [5] A.I. Vainshtein, Phys. Lett. **39B**, 393 (1972) .
- [6] D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
- [7] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. **B485**, 208 (2000) [hep-th/0005016].
- [8] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [hep-th/0010186].
- [9] C. Deffayet, G. Dvali, G. Gabadadze and A. Lue, hep-th/0104201.
- [10] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, Phys. Lett. B **503**, 173 (2001) [hep-th/0011138].
- [11] M. Porrati, Phys. Lett. B **498**, 92 (2001) [hep-th/0011152].
- [12] A. Higuchi, Nucl. Phys. B **282**, 397 (1987); Nucl. Phys. B **325**, 745 (1989).
- [13] I. I. Kogan, S. Mouslopoulos, A. Papazoglou and L. Pilo, hep-th/0105255.
- [14] F. A. Dilkes, M. J. Duff, J. T. Liu and H. Sati, hep-th/0102093;  
M. J. Duff, J. T. Liu and H. Sati, hep-th/0105008.
- [15] M. Fierz, Helv. Phys. Acta **12** (1939) 3;  
M. Fierz, W. Pauli, Proc. Roy. Soc. **173** (1939) 211.
- [16] P. van Nieuwenhuizen, Nucl. Phys. B **60**, 478 (1973).
- [17] G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [hep-th/0008054].
- [18] G. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, hep-ph/0102216.
- [19] C. Deffayet, G. Dvali and G. Gabadadze, astro-ph/0105068.
- [20] R. Dick, hep-th/0105320.
- [21] G. Dautcourt, gr-qc/9603001.
- [22] P. van Nieuwenhuizen, Phys. Rev. D **7** 2300 (1973).